Optimum Noise Measure Configurations for Transistor Negative Resistance Amplifiers

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***Abstract*—A new method, using the noise matrix approach, has been developed for determining the optimum reactive terminations for a transistor employed as a low-noise negative-resistance element in a reflection-mode amplifier. This new method corroborates the less efficient graphical method the authors reported earlier. It is established theoretically and demonstrated numerically that the optimum noise measure of a transistor used in a reflection-mode amplifier is independent of the choice of active terminal and is identical to the optimum noise measure of the same transistor when used in a conventional transmission-mode amplifier.**

***Index Terms*— Low-noise, microwave amplifiers, negative resistance.**

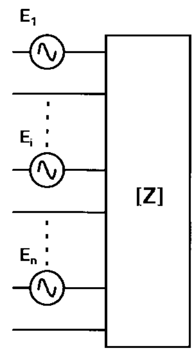

Fig. 1. General n-port noisy network representation.

I. Introduction

NEGATIVE resistance amplifiers, using transistors with appropriate feedback, and terminations, have been reported by several authors [1]–[5]. Possible advantages, compared to conventional transmission amplifiers, include higher gain, subject to a limited gain bandwidth product at high frequencies [3], [5], and the availability of a fail-safe, low-loss bypass path in the case of failure of the device or its power supplies [6].

Negative resistance transistor elements are also becoming increasingly popular for Q-factor enhancement in microwave monolithic integrated circuit (MMIC) filters and resonators, although to date the excess noise arising from the use of active devices in such applications has received little attention. While the analysis given here is primarily directed toward negative resistance amplifier synthesis, it could be extended to enable optimization of noise performance in circuits employing negative resistance Q-factor enhancement.

In an earlier paper [4] the authors showed that an appropriate choice of reactances on two leads of a transistor gives rise to negative resistance and optimum noise measure on the third lead, and the authors presented a graphical method for determining the required reactances. In this paper, a new, more elegant, and efficient analytical method, based on the noise matrix technique of [7], is presented for determining the required reactances. The associated analysis proves that the optimum noise measure is independent of the choice of terminal to be used as the negative resistance terminal, confirming an earlier observation [5].

Furthermore, it is proved that the optimum noise measure of a transistor configured for negative resistance amplification is identical to that of the same transistor when used in a transmission-mode amplifier.

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II. Characteristic Noise Matrix Theory

In order to develop this new method for synthesis of optimum noise measure negative-resistance transistor circuits, it is first necessary to review the concepts of exchangeable noise power and the characteristic noise matrix of active noisy networks, developed by Haus and Adler [8]. This section briefly reviews the relevant theory and sets it in the appropriate context for this application.

A general representation of an n-port noisy active device is given in Fig. 1. It consists of a noiseless n-port device, with a noise voltage source E_i in series with each port. If $\mathbf{V}$, $\mathbf{I}$, and $\mathbf{E}$ are nth-order column vectors of the terminal voltages, the currents flowing into each terminal, and the noise voltages, respectively, then

$$\mathbf{V} = \mathbf{ZI} + \mathbf{E} \tag{1}$$

where $\mathbf{Z}$ is the impedance matrix of the active network. Partial correlation exists between the elements of $\mathbf{E}$, resulting in nonzero off-diagonal elements in the noise power spectral density matrix, $\mathbf{A}$, defined by

$$\mathbf{A} = \overline{\mathbf{EE}^{T*}}. \tag{2}$$

Here, superscript $T*$ indicates the Hermitian conjugate of a matrix and the overbar indicates the time average of a fluctuating quantity.

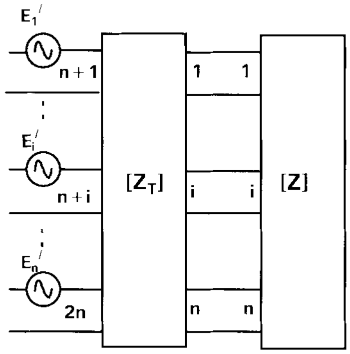

Fig. 2. Transformed n-port noisy network.

Signal and noise properties of microwave FET's or bipolar transistors are conventionally characterized in a two-port common source or common emitter configuration. To select an alternative input port and to add the lossless reactive feedback and terminations required to generate negative resistance at the selected port, a lossless transformation must be introduced. Specific details of the particular transformations needed are given in Section IV.

A general lossless transformation of a noisy n-port network, using a $2n$-port lossless transforming network with impedance matrix $\mathbf{Z}_T$, as shown in Fig. 2, generates a second n-port network, characterized by impedance matrix $\mathbf{Z}'$ and noise voltage vector $\mathbf{E}'$ where

$$\mathbf{Z}' = -\mathbf{Z}_{ba}(\mathbf{Z} + \mathbf{Z}_{aa})^{-1}\mathbf{Z}_{ab} + \mathbf{Z}_{bb} \tag{3}$$

and

$$\mathbf{E}' = \mathbf{Z}_{ba}(\mathbf{Z} + \mathbf{Z}_{aa})^{-1}\mathbf{E}. \tag{4}$$

$\mathbf{Z}_{ab}$, $\mathbf{Z}_{ba}$, $\mathbf{Z}_{aa}$, and $\mathbf{Z}_{bb}$ are submatrices of $\mathbf{Z}_T$, given by

$$Z_{aa} = \begin{pmatrix} Z_{T\,1,1} & \cdots & \cdots & Z_{T\,1,n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ Z_{T\,n,1} & \cdots & \cdots & Z_{T\,n,n} \end{pmatrix}$$

$$Z_{bb} = \begin{pmatrix} Z_{T\,n+1,n+1} & \cdots & \cdots & Z_{T\,n+1,2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ Z_{T\,2n,n+1} & \cdots & \cdots & Z_{T\,2n,2n} \end{pmatrix}$$

$$Z_{ab} = \begin{pmatrix} Z_{T\,1,n+1} & \cdots & \cdots & Z_{T\,1,2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ Z_{T\,n,n+1} & \cdots & \cdots & Z_{T\,n,2n} \end{pmatrix}$$

$$Z_{ba} = \begin{pmatrix} Z_{T\,n+1,1} & \cdots & \cdots & Z_{T\,n+1,n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ Z_{T\,2n,1} & \cdots & \cdots & Z_{T\,2n,n} \end{pmatrix}$$

or, alternatively,

$$Z_T = \left(\begin{array}{c|c} Z_{aa} & Z_{ab} \\ \hline Z_{ba} & Z_{bb} \end{array}\right). \tag{5}$$

In the case of negative resistance amplifiers, the transforming network is lossless but not necessarily reciprocal. This implies that

$$\begin{aligned} \mathbf{Z}_{aa} &= -\mathbf{Z}_{aa}^{T*} \\ \mathbf{Z}_{bb} &= -\mathbf{Z}_{bb}^{T*} \\ \mathbf{Z}_{ab} &= -\mathbf{Z}_{ba}^{T*}. \end{aligned} \tag{6}$$

Ports 1 to n of the transforming network are connected to ports 1 to n of the original noisy network. Ports $n+1$ to $2n$ of the transforming network then become the n ports of the new network.

The exchangeable noise power P_e of one port of a network is defined as the extremum of power output with respect to variations in the port current. For the ith port of an n-port network, with all other ports open circuited, this is given by

$$P_{e\,i} = \frac{\overline{E_i E_i^*}}{2(Z_{ii} + Z_{ii}^*)} \tag{7}$$

where the superscript * indicates the complex conjugate and the overbar indicates a time average.

A one-port negative resistance element can be generated from an n-port active device by imbedding it in a lossless $2n$-port network, containing the necessary reactive terminations and feedback. $n-1$ ports of the resulting n-port network are left open circuited. The exchangeable noise power at the single remaining port is given by

$$P'_{e\,i} = \frac{\overline{E_i' E_i'^*}}{2(Z_{ii}' + Z_{ii}'^*)} = \frac{\xi^T \overline{\mathbf{E}'\mathbf{E}'^{T^*}} \xi}{\mathbf{2}\xi^T(\mathbf{Z}' + \mathbf{Z}'^{T^*})\xi} \tag{8}$$

where ξ is a column matrix used to select the wanted elements in the numerator and denominator of (8), and $\xi_j = 0$ for $j \neq i$, and $\xi_i = 1$.

Substitution of (3) and (4) into (8), and using the lossless network properties expressed in (6), lead to Haus' and Adler's key result [7]

$$P'_{e\,i} = -\frac{\mathbf{x}^{T^*}\mathbf{A}\mathbf{x}}{\mathbf{x}^{T^*}\mathbf{B}\mathbf{x}} \tag{9}$$

where

$$\mathbf{x} = [\mathbf{Z}_{ba}(\mathbf{Z} + \mathbf{Z}_{aa})^{-1}]^{T^*}\xi \tag{10}$$

and

$$\mathbf{B} = -2(\mathbf{Z} + \mathbf{Z}^{\mathbf{T}^*}). \tag{11}$$

$\mathbf{Z}_{ba}$ is unrestricted, as a result of (6), so that the elements of the column matrix $\mathbf{x}$ in (10) can take on all possible complex values, as the transforming network is varied through all possible lossless forms. P'_{ei} can be regarded as a function of the variables x_j and x_j^*. Hence, the stationary values (maxima, minima, or saddle points) of P'_{ei} can be found by setting

$$\frac{\partial P'_{ei}}{\partial x_j} = 0$$

and

$$\frac{\partial P'_{ei}}{\partial x_j^*} = 0, \qquad \text{for all } j. \tag{12}$$

(9) can be rewritten as

$$\mathbf{x}^{T^*}\mathbf{B}\mathbf{x}P'_{ei} + \mathbf{x}^{T^*}\mathbf{A}\mathbf{x} = 0. \tag{13}$$

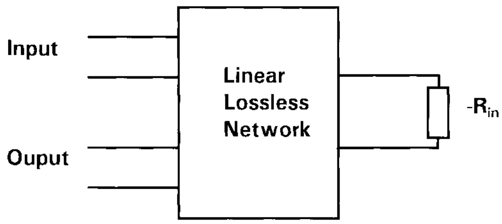

Fig. 3. Negative resistance amplifier configuration.

Differentiating (13) with respect to x_j and x_j^* gives

$$(\mathbf{x}^{T^*}\mathbf{B})_j P'_{e\,i} + (\mathbf{x}^{T^*}\mathbf{A})_j + \mathbf{x}^{T^*}\mathbf{B}\mathbf{x}\frac{\partial P'_{ei}}{\partial x_j} = 0 \quad (14)$$

and

$$(\mathbf{B}\mathbf{x})_j P'_{e\,i} + (\mathbf{A}\mathbf{x})_j + \mathbf{x}^{\mathbf{T}*}\mathbf{B}\mathbf{x}\frac{\partial P'_{ei}}{\partial x_j^*} = 0. \quad (15)$$

Applying the conditions in (12) to (14) and (15) shows that the stationary values of exchangeable power P'_{ei} are solutions of

$$\mathbf{x}^{T^*}\mathbf{B}P'_{ei} + \mathbf{x}^{T^*}\mathbf{A} = 0 \quad (16)$$

and

$$\mathbf{B}\mathbf{x}P'_{ei} + \mathbf{A}\mathbf{x} = 0. \quad (17)$$

Since $\mathbf{A}$ and $\mathbf{B}$ are Hermitian, (16) and (17) are equivalent. Equation (17) can be further rearranged as

$$\mathbf{B}^{-1}\mathbf{A}\mathbf{x} + P'_{ei}\mathbf{x} = 0. \quad (18)$$

It follows that the stationary values of P'_{ei}, with respect to variations in the transforming network, are the negatives of the eigenvalues of the characteristic noise matrix $\mathbf{N}$, where

$$\mathbf{N} = \mathbf{B}^{-1}\mathbf{A} = -\tfrac{1}{2}\,(\mathbf{Z} + \mathbf{Z}^{\mathbf{T}*})^{-1}\,\overline{\mathbf{E}\mathbf{E}^{T^*}}. \quad (19)$$

The transforming network required to achieve a particular stationary value is represented by the corresponding eigenvector.

Section III of the paper explains the connection between the exchangeable noise power and the noise measure, and Section IV shows how the eigenvectors corresponding to optimum noise measure can be used to determine the required element values in the transforming networks.

III. Optimum Noise Measure of Negative Resistance Amplifiers

Penfield [8] showed that the noise measure of an amplifier composed of a one-port negative resistance device connected to a lossless (not necessarily reciprocal) three-port network, as shown in Fig. 3, is given by

$$M = -\frac{P_e}{kT_0B} \quad (20)$$

where P_e is the exchangeable noise power of the one-port, k is Boltzmann's constant, T_0 is the standard reference temperature by convention 290 K, and B is the noise measurement bandwidth. The three-port network is designed to separate the input and output waves and to present the appropriate impedance to the negative resistance element, to achieve the required level of gain without causing oscillations.

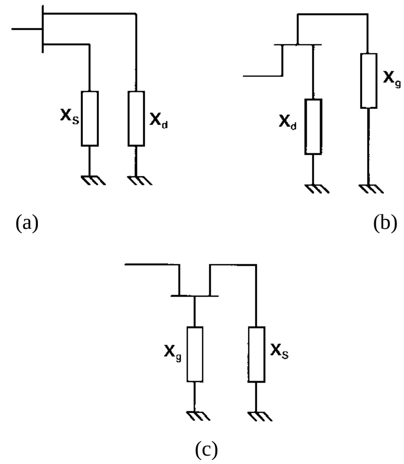

Fig. 4. FET negative resistance element configurations. (a) Gate input. (b) Source input. (c) Drain input.

Penfield's result discussed above shows that the noise measure of the overall amplifier (Fig. 3) depends only on the exchangeable noise power of the negative resistance element, and not on the details of the lossless three-port network. The new analysis presented in Section IV below is concerned with the design of that negative resistance element to achieve the lowest exchangeable noise power and, hence, the optimum noise measure for a given transistor.

IV. Application to Transistor Negative Resistance Amplifiers

As discussed in Section II, a negative resistance one-port element can be generated from an n-port active device by first imbedding it in a transforming network as shown in Fig. 2, and selecting one of the ports as the negative resistance port for use in the network shown in Fig. 3.

If an amplifier has greater than unity gain, it necessarily has $M > 0$. Thus the optimum noise measure of a negative resistance transistor amplifier is given by

$$M = \frac{\lambda}{kT_0B} \quad (21)$$

where λ is the smallest positive eigenvalue of the characteristic noise matrix N (19). This result could also be inferred from the fact that $P_e < 0$ for a negative resistance amplifier (7).

The optimum configuration for the transforming network can be determined by examining the eigenvector corresponding to this minimum positive eigenvalue. The eigenvector is the vector $\mathbf{x}$ which satisfies

$$(\mathbf{N} - \boldsymbol{\lambda}\mathbf{I})\mathbf{x} = 0. \quad (22)$$

In the case of a transistor, the active device is conveniently represented as a two-port, and (22) becomes

$$(N_{1,1} - \lambda)x_1 + N_{1,2}x_2 = 0 \quad (23)$$
$$N_{2,1}x_1 + (N_{22} - \lambda)x_2 = 0 \quad (24)$$

where x_1 and x_2 are related to the reactance values in the specific configurations, as shown in Sections IV-A and IV-B.

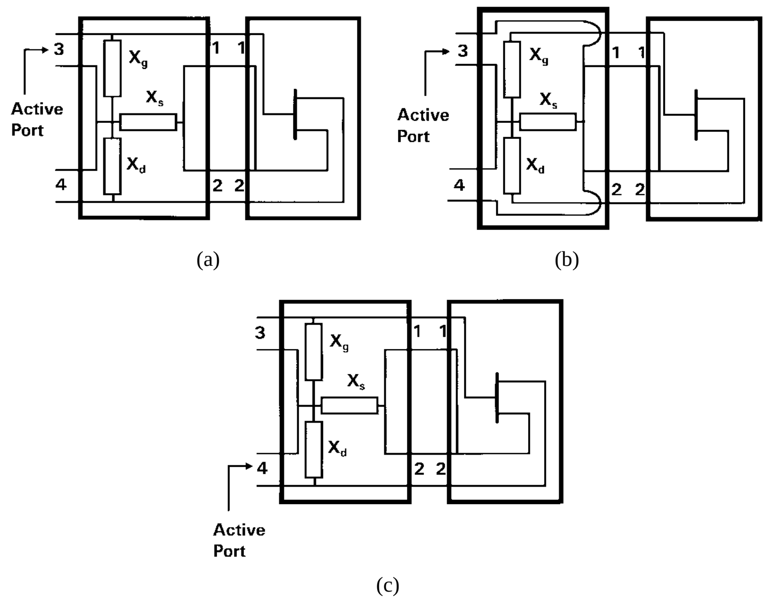

Fig. 5. FET negative resistance configurations in the notation of Fig. 2. (a) Gate input. (b) Source input. (c) Drain input.

Since the eigenvalues are solutions of $|(\mathbf{N}-\boldsymbol{\lambda}\mathbf{I})|=0$, (23) and (24) are linearly dependent. One equation alone is sufficient to define the optimum transformation circuit, i.e.,

$$\frac{x_1}{x_2}=\frac{N_{1,2}}{\lambda - N_{1,1}}=\eta. \tag{25}$$

η, defined by (25), is introduced here to simplify subsequent analysis.

There are three possible configurations for a negative resistance FET element, using the gate, source, or drain, respectively, as the active port. In each case, the other two terminals are reactively terminated. The three configurations are shown in Fig. 4. Separating real and imaginary parts of (25) gives two equations which can be used to find the optimum values of the two terminating reactances.

The three configurations may be represented in terms of the notation used in Fig. 2, regarding the FET as an active two-port network, and incorporating the required terminating reactances into four-port transforming networks, as shown in Fig. 5. To ensure that the elements of the $\mathbf{Z}$ matrices of the transforming networks are finite, additional reactances are required in parallel with the active port. The value of the additional reactance makes no difference to the exchangeable noise power, as it can be tuned out in the external circuit, and, hence, it makes no difference to the noise measure. The resulting transforming networks for the gate and drain input cases [Fig. 5(a) and (c)] are the same. For gate input, port 3 is selected by vector ξ. For drain input, port 4 is selected. In the source input case, a different transforming network [Fig. 5(b)] is required, and ports 3 and 4 are identical, so either can be selected.

The derivations of the optimum terminating reactances can be performed by starting with the $\mathbf{Z}_T$ matrix for each of the three configurations. Section IV-A covers the gate and drain input configurations, which use the same transforming network. Section IV-B covers the source input configuration.

A. Gate and Drain Input Configurations

By inspection of Fig. 5(a) and (c), one can write

$$\mathbf{Z}_T = j\begin{pmatrix} X_g+X_s & X_s & X_g & 0 \\ X_s & X_d+X_s & 0 & X_d \\ X_g & 0 & X_g & 0 \\ 0 & X_d & 0 & X_d \end{pmatrix}. \tag{26}$$

Thus,

$$\mathbf{Z}_{ba} = j\begin{pmatrix} X_g & 0 \\ 0 & X_d \end{pmatrix}$$

and

$$\mathbf{Z}_{aa} = j\begin{pmatrix} X_g+X_s & X_s \\ X_s & X_d+X_s \end{pmatrix}. \tag{27}$$

$$\begin{aligned}\mathbf{x} &= [\mathbf{Z}_{ba}(\mathbf{Z}+\mathbf{Z}_{aa})^{-1}]^{T^*}\boldsymbol{\xi} \\ &= \left\{ j\begin{pmatrix} X_g & 0 \\ 0 & X_d \end{pmatrix}\begin{bmatrix} Z_{11}+j(X_g+X_s) & Z_{12}+jX_s \\ Z_{21}+jX_s & Z_{22}+j(X_d+X_s) \end{bmatrix}^{-1}\right\}^{T^*}\xi \\ &= \frac{1}{\Delta^*}\begin{bmatrix} -jX_gZ_{22}^*-X_g(X_d+X_s) & jX_dZ_{21}^*+X_dX_s \\ jX_gZ_{12}^*+X_gX_s & -jX_dZ_{11}^*-X_d(X_g+X_s) \end{bmatrix}\xi \end{aligned} \tag{28}$$

X can then be determined from (10) as shown in (28) at the bottom of the previous page, where

$$\mathbf{\Delta} = \mathrm{Det}\,(\mathbf{Z} + \mathbf{Z}_{aa}).$$

1) Gate Input Configuration: The gate terminal is selected as the input by putting

$$\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

so that from (28)

$$\frac{x_1}{x_2} = \frac{-jZ_{22}^* - X_d - X_s}{jZ_{12}^* + X_s} = \eta. \tag{29}$$

Note that, as expected, X_g is cancelled from this equation. Separating real and imaginary parts gives

$$X_s = \frac{-R_{22} - R_{12}\,\mathrm{Re}\,\eta}{\mathrm{Im}\,\eta} - X_{12} \tag{30}$$

and

$$X_d = R_{12}\,\mathrm{Im}\,\eta - X_{12}\,\mathrm{Re}\,\eta - X_{22} - (1 + \mathrm{Re}\,\eta)X_s \tag{31}$$

where $X_{ij} = \mathrm{Im}\,Z_{ij}$ and $R_{ij} = \mathrm{Re}\,Z_{ij}$.

2) Drain Input Configuration: The drain terminal is selected as the input by putting

$$\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

so that from (28)

$$\frac{x_1}{x_2} = \frac{jZ_{21}^* + X_s}{-jZ_{11}^* - (X_g + X_s)} = \eta. \tag{32}$$

In this case, X_d cancels as expected. Separating real and imaginary parts as before, one finds

$$X_s = R_{11}\,\mathrm{Im}\,\eta - X_{12} + \frac{\mathrm{Re}\,\eta}{\mathrm{Im}\,\eta}(R_{21} + R_{11}\,\mathrm{Re}\,\eta) \tag{33}$$

and

$$X_g = \frac{-R_{21} - R_{11}\,\mathrm{Re}\,\eta}{\mathrm{Im}\,\eta} - X_{11} - X_s. \tag{34}$$

B. Source Input Configuration

In this case, referring to Fig. 5(b), a different transforming network is required, with a $\mathbf{Z}$ matrix given by

$$\mathbf{Z}_T = j\begin{pmatrix} X_g + X_s & X_S & -X_s & -X_s \\ X_s & X_d + X_s & -X_s & -X_s \\ -X_s & -X_s & X_s & X_s \\ -X_s & -X_s & X_s & X_s \end{pmatrix}. \tag{35}$$

Thus,

$$\mathbf{Z}_{aa} = j\begin{pmatrix} X_g + X_s & X_s \\ X_s & X_d + X_s \end{pmatrix}$$

and

$$\mathbf{Z}_{ba} = -jX_s\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{36}$$

and, X can be determined from (10) as shown in (37), at the bottom of the page. Note that the two columns of the 2 × 2 matrix in (37) are identical. Therefore, it makes no difference which column is selected using the vector ξ. This is because, as previously stated, ports 3 and 4 of the transforming network are identical. In either case, one finds

$$\frac{x_1}{x_2} = \frac{Z_{22}^* - Z_{21}^* - jX_d}{Z_{11}^* - Z_{12}^* - jX_g} = \eta. \tag{38}$$

Again, the reactance in parallel with the chosen port X_s does not appear in the expression for η since it does not affect the noise measure. Separating real and imaginary parts again, one finds

$$X_g = \frac{R_{22} - R_{21} - \mathrm{Re}\,\eta}{\mathrm{Im}\,\eta} - X_{11} + X_{12} \tag{39}$$

and

$$X_d = (X_{11} - X_{12} + X_g)\,\mathrm{Re}\,\eta - (R_{11} - R_{12})\,\mathrm{Im}\,\eta - X_{22} + X_{21}. \tag{40}$$

C. Determination of the Characteristic Noise Matrix

To determine the optimum reactive terminations for a practical microwave negative-resistance transistor amplifier using the method described above, it is first necessary to generate the $\mathbf{Z}$ matrix and the noise power spectral density matrix $\mathbf{A}$ from the S-parameters and noise parameters ($F_{\min}$, $\boldsymbol{\Gamma}_{\mathrm{opt}}$, and R_n) which are more commonly used to characterize microwave two port devices. $\mathbf{Z}$ and $\mathbf{A}$ are then used in (19) to calculate the characteristic noise matrix $\mathbf{N}$.

The $\mathbf{Z}$ matrix is given by (41), shown at the bottom of the following page.By manipulating the equations given in [9], it can be shown that the noise power spectral density matrix $\mathbf{A}$ is given by (42), shown at the bottom of the following page, where

$$Y_{cor} = \frac{F_{\min} - 1}{2R_n} - \frac{1}{Z_0}\frac{1 - \Gamma_{\mathrm{opt}}}{1 + \Gamma_{\mathrm{opt}}}$$

and

$$G_n = (F_{\min} - 1)\,\mathrm{Re}\left(\frac{1}{Z_0}\frac{1 - \Gamma_{\mathrm{opt}}}{1 + \Gamma_{\mathrm{opt}}}\right) - \frac{(F_{\min} - 1)^2}{4R_n}.$$

$$\begin{aligned} \mathbf{x} &= [\mathbf{Z}_{ba}(\mathbf{Z} + \mathbf{Z}_{aa})^{-1}]^{T^*}\boldsymbol{\xi} \\ &= \left[-jX_s\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} Z_{11} + j(X_g + X_s) & Z_{12} + jX_s \\ Z_{21} + jX_s & Z_{22} + j(X_d + X_s) \end{pmatrix}^{-1}\right]^{T^*}\xi \\ &= \frac{jX_s}{\Delta^*}\begin{pmatrix} Z_{22}^* - Z_{21}^* - jX_d & Z_{22}^* - Z_{21}^* - jX_d \\ Z_{11}^* - Z_{12}^* - jX_g & Z_{11}^* - Z_{12}^* - jX_g \end{pmatrix}\xi \end{aligned} \tag{37}$$

D. Optimum Noise Measure and Optimum Reactive Terminations

Thus, for all three configurations, the terminating reactances required for optimum noise measure can be found directly from the characteristic noise matrix $\mathbf{N}$ and the $\mathbf{Z}$ matrix of the active device. $\mathbf{N}$ and $\mathbf{Z}$, in turn, can be determined from the more familiar two-port noise and scattering parameters.

Note that the minimum positive eigenvalue λ is a characteristic of the active device alone. Thus, the optimum noise measure, equal to λ/kT_0B (21), is independent of the configuration of the transforming network, so long as the corresponding eigenvector can be generated. Sections IV-A and IV-B demonstrate that the required eigenvector can be generated in all three configurations. Optimum noise measure in a negative resistance transistor amplifier is, therefore, independent of the choice of active input terminal.

Furthermore, the authors' previous empirical observation [4], that the noise performance achievable in a negative resistance reflection amplifier is comparable to that achievable in a two-port transmission-mode amplifier using the same transistor, is theoretically justified since Haus and Adler [7] showed that optimum noise measure in a two-port transmission-mode amplifier is also equal to λ/kTB. To be precise, the same optimum noise measure is achievable in both cases.

V. Illustrative Example

As an example, the authors demonstrate the application of their method to a commercial high electron-mobility transistor (HEMT) device, type FHX04 FA, at 10 GHz, for which the scattering parameters (referenced to $Z_0 = 50\ \Omega$) and noise parameters are given by the manufacturer as

$$\mathbf{S} = \begin{pmatrix} 0.653\angle -159.8^0 & 0.076\angle -4^0 \\ 2.512\angle 20.2^0 & 0.552\angle -125.7^0 \end{pmatrix}$$

$$\Gamma_{\rm opt} = 0.52\angle 134^0$$

$$F_{\min}\,\text{dB} = 0.66\,\text{dB}$$

$$R_n = 7\,\Omega.$$

Applying the conversions described in (41) and (42), one finds using (19)

$$\mathbf{N} = -\tfrac{1}{2}(\mathbf{Z} + \mathbf{Z}^{T^*})^{-1}\mathbf{A}$$

$$\mathbf{N} = kT_0B\begin{pmatrix} -0.509 + j0.196 & -2.434 - j0.196 \\ -0.037 - j0.048 & 0.117 - j0.196 \end{pmatrix}.$$

The eigenvalues of $\mathbf{N}$ are $\lambda_1 = 0.177kT_0B$ and $\lambda_2 = -0.57kT_0B$

As shown in Section IV, the optimum noise measure corresponds to the smallest positive eigenvalue and, hence, $M_{\rm opt} = \lambda_1/kT_0B = 0.177$.

This gives exact numerical agreement with the transmission-mode optimum-noise measure calculated using the method of [10].

Putting $\lambda = \lambda_1$ in (25), one finds $\eta = -3.200 - j1.201$. For the three negative resistance amplifier configurations, the optimum reactive terminations are then found to be as follows:

Gate input equation (30) and (31):

$$X_d = 45.5\ \Omega$$
$$X_s = 13.2\ \Omega$$

Source input equation (39) and (40):

$$X_g = 53.6\ \Omega$$
$$X_d = -118.2\ \Omega$$

Drain input equation (33) and (34):

$$X_s = 176.9\ \Omega$$
$$X_g = -110.1\ \Omega.$$

Application of the authors' earlier graphical method [4] to this device shows exact numerical agreement with the values of $M_{\rm opt}$ and the terminating reactances for all three cases. The authors' earlier experimental validation of the graphical method [4], therefore, also provides validation of this new, more elegant and efficient matrix method of analysis.

VI. Conclusion

A new analytical technique has been developed to determine the reactive terminations required for low-noise transistor negative-resistance amplifiers with optimum noise measure.

$$\mathbf{Z} = \frac{Z_0}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}\begin{bmatrix} (1+S_{11})(1-S_{22}) + S_{12}S_{21} & 2S_{12} \\ 2S_{21} & (1-S_{11})(1+S_{22}) + S_{12}S_{21} \end{bmatrix} \quad (41)$$

$$\mathbf{A} = 4kT_0B\begin{bmatrix} R_n|1 - Z_{11}Y_{\rm cor}|^2 + G_n|Z_{11}|^2 & -Z_{21}^*Y_{cor}^*R_n + Z_{11}Z_{21}^*(G_n + |Y_{cor}|^2R_n) \\ -Z_{21}Y_{cor}R_n + Z_{11}^*Z_{21}(G_n + |Y_{cor}|^2R_n) & |Z_{21}|^2G_n + |Y_{cor}|^2|Z_{21}|^2R_n \end{bmatrix} \quad (42)$$

The optimum noise measure has been shown theoretically to be invariant with the choice of input terminal, and to have the same value for negative resistance and conventional transmission-mode amplifier configurations. Numerical verifications of this invariance and equivalence have been presented for a commercial HEMT at 10 GHz.

REFERENCES

[1] C. S. Aitchison and K. Madani. "An investigation into the technique of active reactance compensation to improve the gain-bandwidth product performance of microwave bipolar transistor amplifiers," *Proc. Inst. Elect. Eng.,* vol. 127, no. 5, pt. H, pp. 292–300, Oct. 1980.

[2] H. Tohyama and H. Mizuno, "23-GHz band GaAs MESFET reflection type amplifier," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-27, pp. 408–411, May 1979.

[3] S. Nicotra, "13-GHz FET negative resistance half-watt amplifier," in *9th European Microwave Conf. Dig.,* Brighton, U.K., Sept. 1979, pp. 303–307.

[4] P. Gardner and D. K. Paul, "Aspects of the design of low noise, negative resistance, reflection mode transistor amplifiers," *IEEE Trans. Microwave Theory Tech.,* vol. 39, pp. 1869–1875, Nov. 1991.

[5] M. T. Hickson, P. Gardner, and D. K. Paul, "High gain millimetric negative resistance low-noise amplifiers," *IEEE Electron Device Lett.,* vol. 29, p. 1408–1409, Aug. 1993.

[6] P. Gardner and D. K. Paul, "Burnout studies of X-band radar negative resistance transistor low noise amplifiers," *IEEE Electron Device Lett.,* vol. 28, p. 614, July 1992.

[7] H. A. Haus and R. B. Adler, *Circuit Theory of Linear Noisy Networks.* New York: Wiley, 1959.

[8] P. Penfield, Jr, "Noise in negative-resistance amplifiers," *IRE Trans. Circuits Theory,* vol. CT-7, pp. 166–170, June 1960.

[9] H. Rothe and W. Dahlke, "Theory of noisy fourpoles," *Proc. IRE,* vol. 44, pp. 811–818, June 1956.

[10] C. R. Poole and D. K. Paul, "Optimum noise measure terminations for microwave transistor amplifiers," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-33, pp. 1254–1257, Nov. 1985.

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